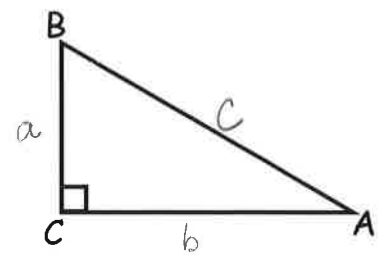
From the Warm-up, we looked at some specific cases that showed $\sin^2 A + \cos^2 A = 1$.

Now we will prove that this identity is true for all right triangles.



Prove: $\sin^2 A + \cos^2 A = \boxed{1}$

$= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$

$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$

$= \frac{a^2+b^2}{c^2}$

By Pyth. Thm., $a^2+b^2=c^2$.

So $\frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = \boxed{1}$ ✓

Basic Identities to KNOW!!!! (The Elite 8)

$\csc\theta = \frac{1}{\sin\theta}$	$\sin\theta = \frac{1}{\csc\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\sec\theta = \frac{1}{\cos\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	
$\cot\theta = \frac{1}{\tan\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Work for all powers. i.e.) $\frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

Now we will use some of the identities to prove a couple more Pythagorean Identities.

2. Given: $\sin^2 A + \cos^2 A = 1$

   Prove: $\tan^2 A + 1 = \sec^2 A$

$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$ (Divide all parts by $\cos^2 A$)

$\tan^2 A + 1 = \sec^2 A$ ✓

3. Given: $\sin^2 A + \cos^2 A = 1$

Prove: $1 + \cot^2 A = \csc^2 A$

$$\frac{\sin^2 A}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A}$$

$$1 + \cot^2 A = \csc^2 A \checkmark$$

Main Three Pythagorean Identities to KNOW!!!

$\sin^2 A + \cos^2 A = 1$ $\tan^2 A + 1 = \sec^2 A$ $1 + \cot^2 A = \csc^2 A$

Now manipulate each one to get two more forms for each.
With that, you'll have a total of 9. (The Starting Line-up)

$1 - \cos^2 A = \sin^2 A$ $\sec^2 A - 1 = \tan^2 A$ $\csc^2 A - 1 = \cot^2 A$

$1 - \sin^2 A = \cos^2 A$ $\sec^2 A - \tan^2 A = 1$ $\csc^2 A - \cot^2 A = 1$

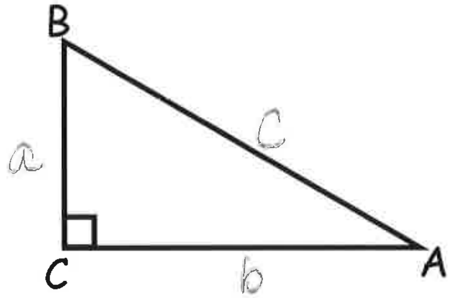
Simplify: (you are not reproving each one; you are using the main three to figure out the answer to each problem)

1. $\cos^2\theta - 1 = -\sin^2\theta$ 2. $\cot^2 B - \csc^2 B = -1$

3. $1 + \tan^2 A = \sec^2 A$ 4. $1 - \sec^2 x = -\tan^2 x$

5. $1 - \sin^2\alpha = \cos^2\alpha$

If time permits, use the triangle to prove two other Pythagorean Identities.



1. Prove: $\sec^2 A - \tan^2 A = \boxed{1}$

$= \left(\frac{c}{b}\right)^2 - \left(\frac{a}{b}\right)^2$

$= \frac{c^2}{b^2} - \frac{a^2}{b^2}$

$= \frac{c^2 - a^2}{b^2}$

Since Pyth. Thm states $a^2+b^2=c^2$, we know $c^2-a^2=b^2$.

So $\frac{c^2-a^2}{b^2} = \frac{b^2}{b^2} = \boxed{1}$ ✓

2. Prove: $1 + \cot^2 A = \boxed{\csc^2 A}$

$1 + \left(\frac{b}{a}\right)^2$

$\frac{a^2}{a^2}\,1 + \frac{b^2}{a^2}$

$\frac{a^2+b^2}{a^2}$

By Pyth. Thm, $a^2+b^2=c^2$.

So $\frac{a^2+b^2}{a^2} = \frac{c^2}{a^2} = \left(\frac{c}{a}\right)^2 = \boxed{\csc^2 A}$ ✓